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Homotopy Analysis Decomposition Method for the Solution of Viscous Boundary Layer Flow Due to a Moving Sheet

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This paper investigates a new approach called Homotopy Analysis Decomposition Method (HADM) for solving nonlinear differential equations, the method was developed by incorporating Adomian polynomial into Homotopy Analysis Method. The Adomian polynomial was used to decompose the nonlinear term in the equation then apply the scheme of homotopy analysis method. The accuracy and efficiency of the proposed method was validated by considering algebraically decaying viscous boundary layer flow due to a moving sheet. Diagonal Pade approximation was used to get the skin friction. The obtained results were presented along with other methods in the literature in tabular form to show the computational efficiency of the new approach. The results were found to agree with those in literature. Owing to its small size of computation, the method is not affected by discretization error as the results are presented in form of polynomials.

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1 Introduction

Fluid is a substance that yields readily to any force that alters its shape; thus, it conforms to the configuration of a containing vessel. Fluids are either in liquid, gaseous or solids form. The study of laminar boundary layer flow of an incompressible fluid has several importance in science and engineering applications such as cooling of an infinite metallic plate in a cooling bath, glass and polymer industries, aerodynamics extrusion of plastic sheets, the boundary layer along liquid film condensation process [1]. The earliest work was dated back to Sakiadis [2] to solve the problem of forced convection along an isothermal constantly moving plate. Since the analytical solutions for these set of nonlinear problems are difficult, an approximate methods for solving them have been getting concentration. Among the methods are Adomian decomposition method [3,4], homotopy analysis method [5,6,7], variational iteration method[8,9], homotopy perturbation method [10,11], weighted residual method [12,13], and their respective modifications have been equally implemented to increase convergent rate or for better understanding. One of our aim is to incorporate the well know Adomian polynomial into homotopy analysis method to have better understanding of HAM that requires repetition of summations when dealing with strongly nonlinear equation, and secondly to check the behavior of the skin friction by using Pade Approximants. Pade approximants as proposed by Henri Pade [14] have largely be used by several authors [1,15,16,17]. It is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function. It often provide better approximation of a function than Taylor Series truncating does, and it may still work in cases where the Taylor Series does not converge. For these reasons, Pade approximants are used extensively in computer calculations and it is now well known that these approximants have the advantage of being able to manipulate polynomial approximation into the rational functions of polynomials. More importantly, the diagonal approximants are the most accurate approximants, therefore we will construct a diagonal approximants throughout this study. Using the boundary condition$f'(\infty) = 0$, the diagonal approximants [M/M] vanish as the coefficients of numerator vanish with the highest power of η. Choosing the coefficients of the highest power of η equal to zero, then solving for the assumed value α using Maple 18 software.

2 Homotopy Analysis Decomposition Method

Consider the scheme [5]

$$U_m(x) = \chi_m U_{m-1}(x) + \hbar \int_0^x \Re_m U_{m-1}(x)dx \quad \hbar \neq 0 \tag{1}$$

$$\chi_m = \begin{cases} 0 & m \leq 1 \\ 1 & m \geq 2 \end{cases} \qquad \Re_m U_{m-1} = \frac{1}{(m-1)!} \frac{\partial^{m-1} N[\phi(x;q)]}{\partial q^{m-1}}\Big|_{\lambda=0} = LU_{m-1} + RU_{m-1} + NU_{m-1}$$

$$\Re_m U_{m-1} = LU_{m-1} + RU_{m-1} + NU_{m-1} \tag{2}$$

Replacing the second term on right hand side of equation (1) by equation (2) gives

$$U_m(x) + \chi_m U_{m-1}(x) + \hbar \int_0^x (LU_{m-1} + RU_{m-1} + NU_{m-1})dx \tag{3}$$

$LU_{m-1}$is the linear operator, $RU_{m-1}$is the remaining of the linear operator, NU_{m-1} is the nonlinear part being replaced with A_{m-1} known as Adomian polynomial and the new scheme was obtained as

$$U_m(x) + \chi_m U_{m-1}(x) + \hbar \int_0^x (LU_{m-1} + RU_{m-1} + A_{m-1})dx \tag{4}$$

Where

$$A_{m-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{d\lambda^{m-1}} \sum_{n=0}^{m-1} (\lambda^n U_n) \Big|_{\lambda=0} \tag{5}$$

The integral in equation (4) depends on the highest derivatives present in the given problem.

3 Application of Homotopy Analysis Decomposition Method

Many analytical and numerical techniques have been described by various authors to solve many boundary layer problems in fluid dynamics. Authors such as Liao applied homotopy analysis method[5], Noor and Mohyud-Din used modified variational iteration method [18], Khan and Faraz applied modified Laplace decomposition method[1], Khan and Smarda also used modified homotopy perturbation transform method[19] for the solution of third order boundary layer equation on semi-infinite domain. Liao [5] presented the semi-infinite boundary layer equation as

$$\begin{gathered} f'''(\eta) + (k-1)f(\eta)f''(\eta) - 2kf'^2(\eta) = 0 \\ f(0) = 0 f'(0) = 1 f'(\infty) = 0 \quad k > 0 \end{gathered} \tag{6}$$

In equation (6), the initial condition at the second derivative is missing which we have to find by representing it as $f''(0) = \alpha$ with the help of Pade approximation.

Using the proposed method, equation (6) gives

$$f_m = \chi_m f_{m-1} + \hbar \int_0^\eta \int_0^\eta \int_0^\eta [f''' + (k-1)ff'' - 2kf'^2] d\eta d\eta d\eta \quad m \geq 1 \tag{7}$$

Identifying the nonlinear parts as

$$A_{m-1} = f_{m-1}f''_{m-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{d\lambda^{m-1}} [\sum_{i=0}^{m-1} (f_i \lambda^i) \sum_{i=0}^{m-1} (f''_i \lambda^i)] \Big|_{\lambda=0} \tag{8}$$

$$B_{m-1} = f'^2_{m-1} = \frac{1}{(m-1)!} \frac{d^{m-1}}{d\lambda^{m-1}} [\sum_{i=0}^{m-1} (f_i \lambda^i)^2] \Big|_{\lambda=0} \tag{9}$$

And decomposing them as

$A_0 = f_0 f''_0$ $\qquad$ $B_0 = f'^2_0$

$A_1 = f_0 f''_1 + f_1 f''_0$ $\qquad$ $B_1 = 2f'_0 f'_1$

$$A_2 = f_0 f_2'' + f_1 f_1'' + f_0 f_2'' \qquad B_2 = 2f_0' f_2' + f_1'^2$$
$$A_3 = f_0 f_3'' + f_1 f_2'' + f_2 f_1'' + f_3 f_0'' \qquad B_3 = 2f_0' f_3' + 2f_2' f_1'$$

From the boundary conditions of equation (6) with assumed constraint $f''(0) = \alpha$, initial approximation was computed using Maclaurin's series as

$$f_0 = \eta + \alpha \frac{\eta^2}{2} \tag{10}$$

Subsequent values of f was generated by repeatedly substituting equations (8 and 9) into equation (7) to obtain

$$f_1 = \frac{1}{3} k\eta^3 + (\frac{1}{8} k\alpha + \frac{1}{24}\alpha)\eta^4 + (\frac{1}{40} k\alpha^2 + \frac{1}{120}\alpha^2)\eta^5 \tag{11}$$

$$f_2 = (\tfrac{1}{30}k + \tfrac{1}{30}k^2)\eta^5 + (\tfrac{1}{240}\alpha + \tfrac{1}{30}k\alpha + \tfrac{19}{720}k^2\alpha)\eta^6 + (\tfrac{11}{5040}\alpha^2 + \tfrac{1}{120}k\alpha^2 + \tfrac{3}{560}k^2\alpha^2)\eta^7 +$$

$$(\tfrac{11}{40320}\alpha^3 + \tfrac{1}{960}k\alpha^3 + \tfrac{3}{4480}k^2\alpha^3)\eta^8 \tag{12}$$

$$f(\eta) = \eta + \alpha\tfrac{\eta^2}{2} + \tfrac{1}{3}k\eta^3 + (\tfrac{1}{8}k\alpha + \tfrac{1}{24}\alpha)\eta^4 + (\tfrac{1}{40}k\alpha^2 + \tfrac{1}{120}\alpha^2 + \tfrac{1}{30}k + \tfrac{1}{30}k^2)\eta^5$$

$$+ (\tfrac{1}{240}\alpha + \tfrac{1}{30}k\alpha + \tfrac{19}{720}k^2\alpha)\eta^6 + (\tfrac{11}{5040}\alpha^2 + \tfrac{1}{120}k\alpha^2 + \tfrac{3}{560}k^2\alpha^2 + \tfrac{2}{315}k^3 + \tfrac{2}{315}k^2 + \tfrac{1}{315}k)\eta^7$$

$$+ (\tfrac{11}{40320}\alpha^3 + \tfrac{1}{960}k\alpha^3 + \tfrac{3}{4480}k^2\alpha^3 + \tfrac{167}{40320}k^3\alpha^2 + \tfrac{33}{4480}k^2\alpha + \tfrac{23}{5760}k\alpha + \tfrac{1}{2688}\alpha)\eta^8$$

$$+ (\tfrac{527}{362880}k^3\alpha^2 + \tfrac{23}{8064}k^2\alpha^2 + \tfrac{709}{362880}k\alpha^2 + \tfrac{13}{22680}k^4 + \tfrac{23}{22680}k^2 + \tfrac{1}{3780}k + \tfrac{19}{11340}\alpha^3 + \tfrac{43}{120960}\alpha^2)\eta^9 ...(13)$$

Equation (13) is the partial solution of equation (6) because it contains the assumed constant α. To obtain the numeric value of α, the condition $f'(\infty) = 0$ was imposed via the diagonal Pade approximation [M/M] that vanish after dividing through with the highest power of η. Equating the coefficient of the highest power to zero, gives a polynomial equation solvable for α for any numeric value of k. The assumed $f''(0) = \alpha$ was then obtained.

Table 1. Comparing the numerical values of $f''(0) = \alpha$ obtained by HADM with MHPTM and MLDM for equation (6)

k	Pade Approximations	HADM	MHPTM[19]	MLDM[1]
0.2	[2/2]	−0.3872983347	−0.3872983347	−0.3872983347
	[3/3]	−0.3821533832	−0.3821533832	−0.3821533832
	[4/4]	−0.3819153845	−0.3819153845	−0.3819153845
	[5/5]	−0.3819148088	−0.3819148088	−0.3819148088
	[6/6]	−0.3819121854	−0.3819121854	−0.3819121854
0.3	[2/2]	−0.5773502692	−0.5773502692	−0.5773502692
	[3/3]	−0.5615999244	−0.5615999244	−0.5615999244
	[4/4]	−0.5614066588	−0.5614066588	−0.5614066588
	[5/5]	−0.5614481405	−0.5614481405	−0.5614481405
	[6/6]	−0.5614419340	−0.5614419340	−0.5614419340
0.4	[2/2]	−0.6451506398	−0.6451506398	−0.6451506398
	[3/3]	−0.6397000575	−0.6397000575	−0.6397000575
	[4/4]	−0.6389732578	−0.6389732578	−0.6389732578
	[5/5]	−0.6389892681	−0.6389892681	−0.6389892681
	[6/6]	−0.6389734794	−0.6389734794	−0.6389734794
0.6	[2/2]	−0.8407967591	−0.8407967591	−0.8407967591
	[3/3]	−0.8393603021	−0.8393603021	−0.8393603021
	[4/4]	−0.8396060478	−0.8396060478	−0.8396060478
	[5/5]	−0.8395875381	−0.8395875381	−0.8395875381
	[6/6]	−0.8396056769	−0.8396056769	−0.8396056769
0.8	[2/2]	−1.007983207	−1.007983207	−1.007983207
	[3/3]	−1.007796981	−1.007796981	−1.007796981
	[4/4]	−1.007646828	−1.007646828	−1.007646828
	[5/5]	−1.007646828	−1.007646828	−1.007646828
	[6/6]	−1.007792100	−1.007792100	−1.007792100

Table 2. Comparison of the numerical value of $f''(0)$ obtained using Pade approximation [3,3] by HADM with MHPTM and HPM with different values of k for equation (6)

k	HADM	HPM [20]	MHPTM [19]
4	-2.500032047	-2.55680000	-2.483954032
10	-4.020346666	-4.04760000	-4.026385103
100	-12.89217544	-12.85010000	-12.84334315
1000	-40.65498834	-40.65560000	-40.65538218
5000	-90.91552836	-90.91270000	-104.8420672

Table 3. The [m,m] HAM and HADM with Pade approximation of $f''(0)$ when $k = \frac{1}{3}$ for different values of m for equation (6)

m	HAM-Pade Approximation[5]	HADM-Pade Approximation
2	-0.5609771	-0.57735026
4	-0.5613269	-0.56145074
6	-0.5616108	-0.56144919
8	-0.5614565	
10	-0.5614484	
12	-0.5614489	
14	-0.5614449	
16	-0.56144923	
18	-0.56144923	
20	-0.56144919	

4 Discussion of Results

Series solution was obtained using the new approach. The obtained solution was subjected to diagonal Pade approximants to handle the boundary condition at infinity in order to get the skin friction as presented in the tables. Table 1 presents the numerical results of the proposed method in comparison with the following methods: Modified Homotopy Perturbation Transform Method (MHPTM) and Modified Laplace Decomposition Method (MLDM). The comparison shows agreement with the aforementioned methods. However, the proposed method has an advantage over MHPTM and MLDM because they have been modified thrice and twice respectively. Also, the proposed method converges faster when each iterations was compared. For higher values of k, Table 2 presents the proposed method in comparison with Homotopy Perturbation Method (HPM) and MHPTM using Pade approximant [3/3]. [5] gave the exact solution at $k = \frac{1}{3}$ to be $f''(0) = -0.56144919$ using Homotopy Analysis-Pade approximation of [20/20] and an exact result was obtained at Pade approximation of [6/6] that give rise to Table 3.

5 Conclusion

In this paper, we have applied homotopy analysis decomposition method to solve non linear boundary layer equation in an unbounded domain.The obtained results were compared with other methods to validate the accuracy of the new approach as shown in the results. The new approach (HADM) is also valid for other nonlinear differential equations.

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Competing Interests

Authors have declared that no competing interests exist.

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