BOWEN UNIVERSITY
COLLEGE OF SOCIAL AND MANAGEMENT SCIENCES
ECONOMICS PROGRAMME
B.Sc. DEGREE EXAMINATION 2020/2021 ACADEMIC SESSION
FIRST SEMESTER EXAMINATION

Course code: ECN 203 **Time Allowed: 2 hours 30 mins**
Course Title: Mathematical Techniques for Economists I **Course credit: 3**
Instruction: Provide your answers clearly and show your workings step by step.

Section A: You are required to attempt all in this section

1. Suppose the sales revenue (S) of a firm depends on the quality of its advertisement (A). The functional relationship is expressed as $S = 14 + 16A - 2A^2$. Find the value of A which maximises S.
2. Find the value of Marginal Product of Capital and Labour (MP_K and MP_L) given that output $Q = 5K^{0.4}L^{0.5}$
3. The determinant used to test for functional dependence in both linear and non-linear equations is .... The determinant is composed of all..of a system arranged in an ordered sequence.
4. Determine the elasticity of demand, E_d, if the demand function is $Q = 200 - 4P$ when $P = 20$.
5. Find the Zx, Zxy, Zy and Zyx when Z = (3x + 5) (2x 6y).
6. Transpose these matrices and indicate their new order

   a. $\begin{bmatrix} 6 & 7 & 9 \\ 2 & 8 & 4 \end{bmatrix}$

   b. $\begin{bmatrix} 12 \\ 19 \\ 25 \end{bmatrix}$
7. If $y = 10\sqrt{x} - \frac{9}{x} + \frac{18}{x}$. Find dy/dx
8. Maximise the total revenue function $TR = 32Q - Q^4$.
9. a. What is a Square matrix?
   b. A zero matrix is also known as ....
10. Consider the cost function $C(q) = 3q^2 + 2q + 5$

    (a). What is the marginal cost?

    (b). Is the cost function a concave or a convex?

11. If $A = \begin{matrix} 1 & 3 & 4 \\ -2 & 4 & 8 \\ 3 & -2 & -1 \end{matrix}$ and $B = \begin{matrix} 4 & 1 & 0 \\ 1 & 3 & 5 \\ 0 & 1 & 6 \end{matrix}$. Find A + B.

12. $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 \\ 2 & 3. \\ 5 & 0 \end{bmatrix}$ Find C when C = AB

13. A firm faces a demand curve $P = 17 - 3Q$. find an expression for the TR and MR *in terms of Q*.

14. Given $a_{21} = 4$, $a_{32} = 5$, $a_{13} = 3$, $a_{23} = 6$, $a_{12} = 10$, and $a31 = -5$, complete the matrix below

$$A = \begin{bmatrix} 6 & - & - \\ - & 7 & - \\ - & - & 9 \end{bmatrix}$$

15. Express this system of linear equation in matrix form

$7 X_1 + 3 X_2 = 45$

$4 X_1 + 5 X_1 = 29$

SECTION B: Answer any two questions in this section

1. Given the following functions: $Z = 3x_1^2 - 5x_1 - x_1 x_2 + 6 x_2^2 - 4x_2 - 2x_2x_3 + 4x_3^2 + 2x_3 - 3x_1x_3$

   $Z = -5x_1^2 + 10x_1 + x_1 x_3 - 2x_2^2 + 4x_2 + 2x_2x_3 - 4x_3^2$

   a. Find the critical values of x_1, x_2 and x_3 using crammers rule for the First Order condition
   b. Use the Hessian determinant for the Second Order Condition. (12 marks)

   c. Determine the marginal and Average function for each of the following total function. Evaluate each at Q = 3

   i. $TC = 3Q^2 + 7Q + 12$

   ii. $TR = 12Q - Q^2$ (8 marks)

2. Supposing we have two complementary goods X and Y, whose demand and supply functions are given as

   $Qd_x = 205 - 2.5 P_x - P_y$
   $Qs_x = -30 + 1.5 P_x$
   $Qd_y = 147.5 - 0.5 P_x - 1.5P_y$
   $Qs_y = - 60 + P_y$

   a. Find the equilibrium price and quantity for X and Y using the matrix method. (12 marks)
   b. Find the derivative of the following
      i. $g(x) = 3x^2 + 2 - 8x^{-1/4}$
      ii. $y(x) = 12x^4 + 7x^3 - 4x^2 - 2x + 8$
      iii. $y = \frac{3x^4}{4x^2 + 5}$
      iv. $y = e^{-7x}$ (8 marks)

3. a. Assume that a firm producing a single product in a two distinct markets has the demand functions

   $Q_1 = 21 - 0.1P_1$
   $Q_2 = 50 - 0.4P_2$

   and the joint cost function is

   $TC = 2000 + 10Q$ where $Q = Q_1 + Q_2$

i. What is the profit-maximising level of output and price in the market with discrimination?
ii. What is the profit-maximising level of output and price in the market with discrimination?
iii. What is the profit differential in the two markets? (14 marks)

b. Test for the functional dependence in each of the following by means of the Jacobian

i. $y_1 = 4x_1 - x_2$
ii. $y_2 = 16x_1^2 - 8x_1x_2 + x_2^2$ (6 marks)

4. Use the Lagrangian multiplier to solve the following constrained optimisation problems.

a. Given a budget constraint of $108 when the Price of K = 3 and L = 4, if the generalized Cobb-Douglas production function is $Q = K^{0.4}L^{0.5}$.

i. Find the value of L, K and λ.
ii. Obtain the optimal value of Q
iii. Estimate the effect on the value of the objective function from a 1-unit decrease in the constant of the constraint. (10 marks)

b. What output mix should a maximising firm produce when its total profit function is $\pi = 80x - 2x^2 - xy - 3y^2 + 100y$ and its maximum output capacity is $x + y = 12$. Estimate the effect on profits if output is expanded by 1 unit. (10 marks)