BOWEN UNIVERSITY, IWO, OSUN STATE, NIGERIA
DEPARTMENT OF PHYSICS AND SOLAR ENERGY
2014/2015 SESSION FIRST SEMESTER EXAMINATIONS
PHY 461 - QUANTUM MECHANICS 1

26th January 2015 TIME: 8:30 am - 10:30 am

ANSWER ANY THREE QUESTIONS

1. Consider a two state system with normalized states given by

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad |2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

The Hamiltonian H and another operator U are given by

$$H = \alpha \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

where α is positive.

(a) Find the eigenvalues of H.

(b) Find the normalized eigenstates of H.

(c) What is the probability that a measurement of U will be zero if the system is in the lower energy state?

(d) What is the expectation value of U for the system in the lower energy state?

2. (a) Show that the 2×2 matrices

$$S_1 = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad S_2 = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad S_3 = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

satisfy the commutation relations for angular momentum ($[S_1, S_2] = i\hbar S_3$ and cyclic permutations).

(b) Show that there are two linearly independent eigenvectors $\psi_\pm$ of S_3 with eigenvalues $\pm s\hbar$, where s is a number that you should determine.

(c) Show that the vectors $\psi_\pm$ are also eigenvectors of $S^2 = S_1^2 + S_2^2 + S_3^2$, and that they both have eigenvalue $s(s+1)\hbar^2$.

3. Two non-interacting particles with the same mass m, are in a one-dimensional potential which is zero along a length $2a$, and infinite elsewhere. The single particle space wavefunctions have the form

$$\psi_n(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{n\pi x}{2a}\right),$$

where n is a positive integer, with corresponding energy

$$E_n = n^2 \frac{\pi^2\hbar^2}{8ma^2} = n^2 E_0.$$

(a) What are the values of the three lowest energies of the system?

(b) What are the degeneracies of these energies if the two particles are distinguishable spin-1/2 fermions?

4. Consider a physical system whose Hamiltonian, H, in a two-dimensional Hilbert space is given by

$$H = a\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

where a and b are positive constants, with $a \gg b$.

(a) Use perturbation theory to find the ground state energy of the system, to second order in b.

(b) Find the exact eigenenergies E_0 and E_1 of H.

(c) Expand your result for E_0 in question (b) in a Maclaurin series in b/a. What deduction can you make?