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Abstract

The flow of a viscous incompressible fluid between two parallel plates due to the normal motion of the plates for the two dimensional flow is considered. The governing nonlinear equations and their associated boundary conditions are transformed into a nonlinear ordinary differential equation. The solution of this problem is obtained by Kamal decomposition transform method. Graphical results are presented to investigate the influence of the squeeze number. The validity of our solution is verified by approximate analytical results obtained by homotopy perturbation method (HPM).

Keywords: Kamal transform method, Adomian polynomial,Two dimensional unsteady flow

1. INTRODUCTION

The fluid flow between the two parallel surfaces has received much attention because of its usefulness in the field of science and engineering. The squeezing flow between the parallel walls or plates occurs a lot in industrial applications like a moving piston in engines, hydraulic brakes, and many other devices are based on the principle of flow between contracting domains [1]. Many researchers have used different methods to solve the nonlinear equation governing the two dimensional unsteady flow, among them are homotopy analysis method [2], homotopy perturbation method [4], spline collocation method [1] and weighted residual method [3] to obtain the approximate solutions.

In line with the discussions above, the purpose of this paper is to examine the problem of two dimensional unsteady flows due to the expansion or contraction of parallel plates analytically. The governing equation here is a highly nonlinear equation, which is solved by using the Kamal decomposition transform method.

2. PROBLEM FORMULATION

Let the position of the two plates be given at $z = \pm l(1-\alpha t)^{1/2}$ as shown in figure 1. where l is the position at time $t = 0$ for any two dimensional case, we assumed that the length is 1. D (in the axisymmetric case) is the diameter which is more larger than $2z$ at any time, α is a positive constant, and the two plates are squeezed until they touch each other at $t = 1/\alpha$. When α is negative, the two plates separated. Let u, v, and w be the velocity components in the x, y, and z-direction respectively. For two dimensional flow, Wang introduced the following transform [5].

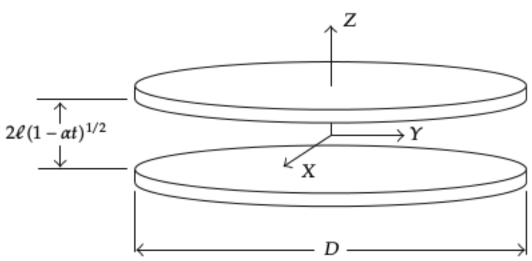

Figure 1: Schematic diagram of the problem.

$$u = \frac{ax}{(4(1-\alpha t))} f'(\eta), \tag{1}$$

$$w = \frac{-\alpha l}{(2(1-\alpha t)^{1/2})} f(\eta), \tag{2}$$

where,

$$\eta = \frac{z}{(l(1-\alpha t)^{1/2})}. \tag{3}$$

By substituting equation (1), and equation (2) into the unsteady two dimensional Navier-Stokes equation gives the nonlinear ordinary differential equation in the form

$$f'''' + S\left(-\eta f''' - 3f'' - \beta f' f'' + f f'''\right) = 0, \tag{4}$$

where $S = al^2/2v$ (squeeze number) is the non-dimensional parameter with the boundary conditions. The flow is characterized by this parameter. Then the boundary conditions are such that on the plates the lateral velocities are zero and the normal velocity is equal to the velocity of the plates, that is,

$$f(0) = 0, \;\; f''(0) = 0, \;\; f(1) = 1, \;\; f'(1) = 0. \tag{5}$$

where

$$\beta = \begin{cases} 0, & \text{axisymmetric,} \\ 1, & \text{two dimensional,} \end{cases}$$

are subjected to boundary conditions in (5).

3. PROPERTIES OF KAMAL TRANSFORM AND ITS DERIVATIVES

Kamal transform is defined for the function of exponential order [6]. Consider the functions in the set S defined by

$$S = \left\{ f(t) : k_1, k_2 > 0, |f(t)| < Me^{|t|/k_j}, t \in (-1)^j \times [0, \infty) \right\}, \tag{6}$$

where M is a constant which must be a finite number, k_1, k_2 are also constants which can be finite or infinite. Kamal transform is denoted by the operator $K(\cdot)$ which is defined as:

$$K\left[f(t)\right] = \int_0^\infty f(t)e^{-t/v}dt = G(v). \quad t \geq 0, \quad k_1 \leq v \leq k_2. \tag{7}$$

Theorem 1. *Let $G(v)$ be a Kamal transform of $f(t)$ where $K[f(t)] = G(v)$ then*

1. $K[f'(t)] = \frac{1}{v}G(v) - f(0)$,
2. $K[f''(t)] = \frac{1}{v^2}G(v) - \frac{1}{v}f(0) - f'(0)$,
3. $K[f^n(t)] = \frac{1}{v^n}G(v) - \sum\limits_{k=0}^{n-1} v^{k-n+1} f^k(0)$.

3.1. BASIC IDEA OF KAMAL-ADOMIAN DECOMPOSITION METHOD

According to [7], the general form of a nonlinear non-homogeneous partial differential equation can be considered as follows

$$Du(x,t) + Ru(x,t) + Nu(x,t) = f(x,t), \tag{8}$$

with the initial conditions $u(x,t) = h(x)$, and $u_t(x,0) = g(t)$, where D is the second-order linear differential operator, R is the linear differential operator of order less than D, N represents the general nonlinear differential operator and $f(x,t)$ is the source term.

By applying the Kamal transform on equation (8), we get

$$K[Du(x,t)] + K[Ru(x,t)] + K[Nu(x,t)] = K[f(x,t)]. \tag{9}$$

Using the differential property of Kamal transform and the given initial conditions, this yields

$$K[u(x,t)] = v^2K[f(x,t)] + vh(x) + v^2g(x) - v^2K[Ru(x,t) + Nu(x,t)]. \tag{10}$$

Applying the inverse Kamal transform on equation (10) gives

$$u(x,t) = G(x,t) - K^{-1}\left[v^2K[Ru(x,t) + Nu(x,t)]\right], \tag{11}$$

where $G(x,t)$ denotes the term arising from the source term and the prescribed initial conditions. Applying the Adomian decomposition method given in [8] into (11) gives

$$u(x,t) = \sum_{n=0}^{\infty} u_n(x,t). \tag{12}$$

And the nonlinear term is decomposed as

$$Nu(x,t) = \sum_{n=0}^{\infty} A_n(u), \tag{13}$$

where A_n are the Adomian polynomials [9] which can be computed using the relation

$$A_n = \frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}\left[N\left(\sum_{n=0}^{\infty}\lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, 3, \cdots \tag{14}$$

Substituting equations (12) and (13) into equation (11) gives

$$\sum_{n=0}^{\infty} u_n(x,t) = G(x,t) - K^{-1}\left[v^2K\left[\sum_{n=0}^{\infty} u_n(x,t) + \sum_{n=0}^{\infty} A_n(u)\right]\right]. \tag{15}$$

From equation (15)

$$u_0(x,t) = G(x,t),$$

and the recursive relation is given by

$$u_{n+1}(x,t) = -K^{-1}\left[v^2K\left[Ru_n(x,t) + A_n(u)\right]\right].$$

In conclusion, after applying Kamal transform and the inverse Kamal transform, we get u_0, u_1, $u_2, \cdots, u_n$ which are the series form of the desired solutions.

4. APPLICATION

The Kamal transform described above is applied to solve the nonlinear equation governing the two dimensional flow given by

$$f'''' + S\left(-\eta f''' - 3f'' - f'f'' + ff'''\right) = 0, \tag{16}$$

with the initial conditions

$$f(0) = 0, \;\; f''(0) = 0, \;\; f(1) = 1.$$

Applying the Kamal transform on equation (16) and applying the given initial conditions, we get

$$K(f) = av^2 + bv^4 - v^4SK\left[-\eta f''' - 3f'' - f'f'' + ff'''\right]. \tag{17}$$

Applying the inverse Kamal transform to both sides of equation (17) yields

$$f(\eta) = a\eta + \frac{b\eta^3}{6} - SK^{-1}\left[v^4K\left[-\eta f'''(\eta) - 3f''(\eta) - f'(\eta)f''(\eta) + ff'''(\eta)\right]\right]. \tag{18}$$

From equation (18), let

$$f_0(\eta) = a\eta + \frac{b\eta^3}{6}.$$

Then, the recursive relation is given as

$$f_{n+1}(\eta) = -SK^{-1}\left[v^4K\left[-\eta f_n'''(\eta) - 3f_n''(\eta) - A_{n_1} + A_{n_2}\right]\right], \tag{19}$$

where A_{n_1} and A_{n_2} are the Adomian polynomials that represent nonlinear term in the equation.

By recursive relation,we have the following series solutions

$$
\begin{aligned}
f_0(\eta) &= a\eta + \frac{1}{6}b\eta^3, \\
f_1(\eta) &= \frac{1}{30}\eta^5 Sb - \frac{1}{120}\eta^5 Sab - \frac{1}{5040}\eta^7 Sb^2, \\
f_2(\eta) &= \frac{1}{120}\eta^7 S^2 b - \frac{1}{280}\eta^7 S^2 ab - \frac{13}{90720}\eta^9 S^2 b^2 + \frac{1}{1680}\eta^7 a^2 b S^2 \\
&+ \frac{1}{22680}\eta^9 ab^2 S^2 + \frac{1}{1108800}\eta^{11} b^3 S^2, \\
f_3(\eta) &= \frac{1}{1890}\eta^9 S^3 b - \frac{43}{997920}\eta^{11} S^3 b^2 + \frac{5}{5660928}\eta^{13} b^3 S^3 \\
&- \frac{1051}{217945728000}\eta^{15} b^4 S^3 - \frac{11}{15120}\eta^9 S^3 ab + \frac{19}{60480}\eta^9 S^3 a^2 b \\
&+ \frac{67}{1995840}\eta^{11} S^3 ab^2 - \frac{1}{24192}\eta^9 S^3 a^3 b - \frac{241}{39916800}\eta^{11} S^3 a^2 b^2 \\
&- \frac{71}{239500800}\eta^{13} S^3 ab^3,
\end{aligned}
$$

$$
\begin{aligned}
f_4(\eta) &= \frac{1}{20790}\eta^{11} S^4 b - \frac{547}{64864800}\eta^{13} S^4 b^2 + \frac{60737}{163459296000}\eta^{15} S^4 b^3 \\
&- \frac{83}{831600}\eta^{11} S^4 ab + \frac{23}{2059200}\eta^{13} S^4 ab^2 + \frac{2243}{1140023808000}\eta^{17} S^4 ab^4 \\
&+ \frac{1}{380160}\eta^{11} S^4 a^4 b + \frac{131}{207567360}\eta^{13} S^4 a^3 b^2 \\
&+ \frac{1357}{25147584000}\eta^{15} S^4 a^2 b^3 - \frac{10429}{1852538688000}\eta^{17} S^4 b^4 \\
&+ \frac{21919}{844757641728000}\eta^{19} S^4 b^5 - \frac{79}{3326400}\eta^{11} S^4 a^3 b \\
&+ \frac{149}{31449600}\eta^{13} S^4 a^2 b^2 - \frac{95159}{3269185920000}\eta^{15} S^4 ab^3 \\
&+ \cdots
\end{aligned}
$$

The iteration is truncated at f_4 and we have the series solution;

$$f = f_0(\eta) + f_1(\eta) + f_2(\eta) + f_3(\eta) + f_4(\eta) + \cdots$$

Imposing the boundary condition $f(1) = 1$ and $f'(1) = 0$ to obtain the constants a and b and using different values of squeeze number (S) and different values of η as in [5] and [3], then the following results are obtained.

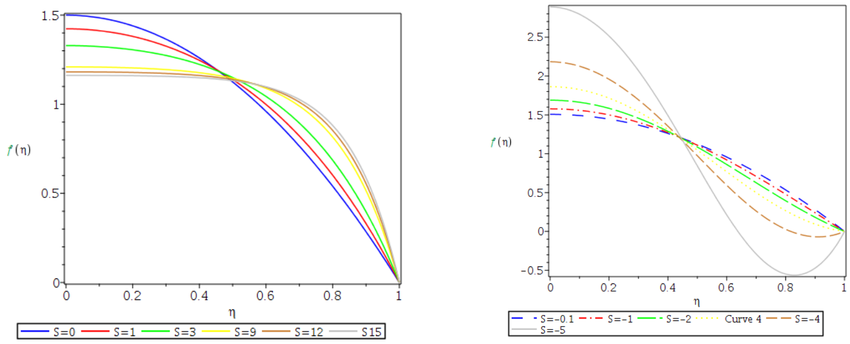

Figure 2: The left panel shows the effect of positive squeeze number (S) on two dimensional unsteady flow, while the influence of negative squeeze number (S) on two dimensional unsteady flow is shown on the right panel.

Table 1: The comparison of analytical solution of $f(\eta)$ at different term of approximation with the numerical solution of homotopy perturbation method (HPM), Runge-kuta method (RK4) and Kamal decomposition transform method (KDTM) for two dimensional unsteady flow.

S	η	HPM	RK4	KDTM
-1.5	0.2	0.333617	0.333618	0.333619
	0.4	0.624358	0.624358	0.624358
	0.6	0.839324	0.839324	0.839324
	0.8	0.962983	0.962983	0.962985
-0.5	0.2	0.305545	0.305545	0.305545
	0.4	0.582470	0.582470	0.582470
	0.6	0.804392	0.804392	0.804392
	0.8	0.949108	0.949108	0.949108
0.5	0.2	0.288260	0.288260	0.288260
	0.4	0.556143	0.556143	0.556143
	0.6	0.781671	0.781671	0.781671
	0.8	0.939640	0.939640	0.939640
1.5	0.2	0.276432	0.276432	0.276432
	0.4	0.537752	0.537752	0.537752
	0.6	0.765249	0.765249	0.765249
	0.8	0.932471	0.932471	0.932471

5. CONCLUSION

In this paper, we applied Kamal transform and Adomian polynomial to solve aforementioned equation approximately. The problem solve showed that the method is very powerful to handle some nonlinear equations efficiently because of its accuracy as compared with other methods.

Figure 2 shows the effect of positive and negative squeeze number (S) on two dimensional unsteady flow, and table 1 shows that the solution obtained happened to be in agreement with that of homotopy perturbation method (HPM) and Runge Kutta method (RK4). Solving some other nonlinear differential equations is very easy using this method.

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