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Weighted Residual Method for the Squeezing Flow between Parallel Walls or Plates

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Abstract: *The nonlinear governing equation is transformed into a fourth order non-linear ordinary differential equation of a viscous incompressible fluid flow between two parallel plates due to the normal motion of the plates for two cases. Its two cases of two-dimensional flow and the axisymmetric flows were examined. The method of weighted residual was used to get the numerical solution and the results thereby presented in tabular form in comparison with existing literature (Runge-Kutta) at a specified region. Various results are graphically shown to demonstrate the effect of (positive and negative) squeeze number on the velocity to check the efficiency and accuracy of the method.*

Keywords: *Weighted Residual Method; Runge-Kutta; Squeezing flow; Axisymmetric flow; Two-dimensional flow.*

Introduction

Fluid flow between two parallel surfaces has received much attention because of its relevance in sciences and engineering. Squeezing flow between parallel walls or plates occurs a lot in industrial application like moving piston in engines, hydraulic brakes and many other devices are based on the principle of flow between contracting domains [1]. Many numerical methods have been used like homotopy analysis method [2], homotopy perturbation method [3], and Spline Collocation method [1] to get the approximate solution.

I. Mathematical Formulation

Let the position of the two plates be at $z=\pm l(1-\alpha t)^{\frac{1}{2}}$, where l is the position at time $t=0$ as shown in figure 1. We assume that the length l (in the two-dimensional case) or the diameter D (in the axisymmetric case) is much larger than the gap width $2z$ at any time such that the end effects can be neglected. When α is negative, the two plates are separated and if α is positive, the two plates are squeezed until they touch at $t={1}/{\alpha}$. Let u, v and w be the velocity components in the x y, and z directions, respectively. For two-dimensional flow, Wang introduced the following transforms [4]

$$u=\frac{\alpha x}{[2(1-\alpha t)]}f'(\eta),\ w=\frac{-\alpha l}{\left[2(1-\alpha t)^{1/2}\right]}f(\eta) \tag{1}$$

Where $$\eta=\frac{z}{\left[l(1-\alpha t)^{1/2}\right]} \tag{2}$$

Substituting equation (1) into unsteady two dimensional Navier-Stokes equation yields nonlinear ordinary differential equation

$$f^{iv}+S\{-\eta f'''-3f''-f'f''+ff'''\}=0, \tag{3}$$

Where squeeze number $S = \frac{\alpha l^2}{2\nu}$ is the non-dimensional parameter under which the flow is characterised, ν is the kinematic viscosity. The boundary conditions are such that on the plates the lateral velocities are zero and the normal velocity is equal to the velocity of the plate, that is,

$$f(0)=0,\ f''(0)=0,\ f(1)=1,\ f'(1)=0 \tag{4}$$

Also, Wang's transform for axisymmetric flow [4]

$$u = \frac{\alpha x}{[4(1-\alpha t)]} f'(\eta),\ v = \frac{\alpha y}{[4(1-\alpha t)]} f'(\eta),\ w = \frac{-\alpha l}{\left[2(1-\alpha t)^{1/2}\right]} f(\eta) \tag{5}$$

Using equation (5), unsteady axisymmetric Navier-Stokes equation reduce to

$$f^{iv} + S\{-\eta f''' - 3f'' + ff'''\} = 0 \tag{6}$$

Subject to boundary condition of equation (4)
Consequently, we solve the nonlinear ordinary differential equation

$$f^{iv}(\eta) + S\left[-\eta f'''(\eta) - 3f''(\eta) - \beta f'(\eta) f''(\eta) + f(\eta) f'''(\eta)\right] = 0 \tag{7}$$

$$f(0)=0,\ f''(0)=0,\ f(1)=1,\ f'(1)=0$$

$$\beta = \begin{cases} 0, & axisymmetric, \\ 1, & two\text{-}\dim ensional \end{cases}$$

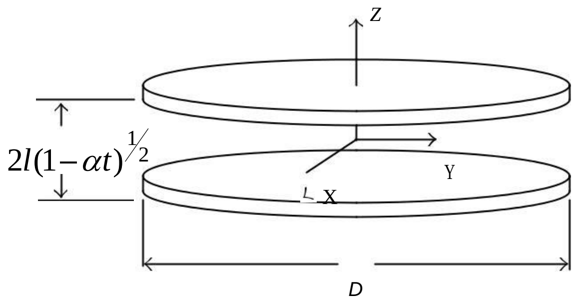

Figure 1: Schematic diagram of the problem.

II. Weighted Residual Method

Weighted residual method have been studied by many researchers to investigate the solution of many nonlinear problems such as MHD Jeffery-Hamel flow [5], flow of viscoelastic fluid over a stretching sheet [6], Bratu-type problem [7], Problems with semi-infinite domain [8] and we are using the same method to solve fluid problem of fourth order.

We seek for a polynomial of the form

$$w(\eta, a) = \phi_0(\eta) + \sum_{i=1}^{N} a_i \phi_i(\eta) \tag{9}$$

where $\phi_0(\eta)$ satisfies the given boundary conditions and each $\phi_i(\eta)$ satisfies the homogenous form of the boundary conditions. The function $w(\eta, a)$ is then used as an approximation to the exact solution in the equation

$$L(U(\eta)) = Q(\eta) \tag{10}$$

to give

$$R(\eta) = L(U(\eta)) - Q(\eta) \tag{11}$$

The function $R(\eta)$ is the residual. The idea is to make $R(\eta)$ as small as possible. One of the methods of minimizing $R(\eta)$ is Galerkin method which was used in this work. $R(\eta)$ is set to zero at some points in the interval. The system of these equations is then solved to determine the parameters a_i . $w(\eta, a)$ is then considered as the approximate solution. Any polynomial can be used provided it satisfies the conditions of equation (7).

III. Application of Weighted Residual Method

Forcing the trial function

$$F(\eta) = \sum_{i=0}^{N} a_i \eta^i \tag{12}$$

to satisfy the boundary condition of equation (7) and substituting equation (12) into equation (7), gives the residual.

For $N = 13, S = -0.5, \beta = 0$, we have the solution as

$$\begin{aligned} F(\eta) &= 1.535702961\eta - 0.5712593134\eta^3 + 0.03521724945\eta^5 \\ &-0.0002645873264\eta^6 + 0.001843382027\eta^7 - 0.003338395464\eta^8 \\ &+0.005168645285\eta^9 - 0.005454162841\eta^{10} + 0.003387547401\eta^{11} \\ &-0.001183239062\eta^{12} + 0.0001799583891\eta^{13} \end{aligned} \tag{13}$$

At $\eta = 0.2$, equation (13) gives 0.3025817785 which is in agreement with [Saeed and Abed (2012)] 0.302582.

For $N = 13, S = -0.5, \beta = 1$, we have the solution as

$$\begin{aligned} F(\eta) &= 1.551956301\eta - 0.6082453274\eta^3 - 0.2137224300\eta^5 \\ &+3.185610125\eta^6 - 15.51982851\eta^7 + 41.77568013\eta^8 - 68.42991066\eta^9 \\ &+70.14371624\eta^{10} - 44.09142112\eta^{11} + 15.58139280\eta^{12} - 2.375227547\eta^{13} \end{aligned} \tag{14}$$

At $\eta = 0.2$, equation (14) gives 0.3055403828 which is also in agreement with [Saeed and Abed (2012)] 0.305545.

For specified value of $N = 13$, we have system of equations where the values of $a's$ were determined for different values of S as presented in tables 1 and 2 for axisymmetric and two-dimensional cases respectively.

Table 1: The analytic results of $f(\eta)$ at different order of approximation in comparison with Runge-Kutta of fourth order for the axisymmetric case [2]

S	η	Numerical	WRM
-1.5	0.2	0.319526	0.319727
	0.4	0.603830	0.603994
	0.6	0.822876	0.822736
	0.8	0.956801	0.956886
-0.5	0.2	0.302582	0.302582
	0.4	0.578082	0.578082
	0.6	0.800780	0.800780
	0.8	0.947702	0.947702
0.5	0.2	0.290322	0.290321
	0.4	0.559252	0.559254
	0.6	0.784303	0.784302
	0.8	0.940703	0.940703
1.5	0.2	0.281010	0.281218
	0.4	0.544779	0.545115
	0.6	0.771371	0.770415
	0.8	0.935036	0.935234
2.5	0.2	0.273682	0.273682
	0.4	0.533246	0.533246
	0.6	0.760847	0.760847
	0.8	0.930280	0.930280

Table 2: The analytic results of $f(\eta)$ at different order of approximation in comparison with Runge-Kutta of fourth order for the two-dimensional case [2]

S	η	Numerical	WRM
-1.5	0.2	0.333618	0.333619
	0.4	0.624358	0.624357
	0.6	0.839325	0.839326
	0.8	0.962984	0.962984
-0.5	0.2	0.305545	0.305540
	0.4	0.582470	0.582477
	0.6	0.804392	0.804386
	0.8	0.949108	0.949110
0.5	0.2	0.288260	0.288260
	0.4	0.556143	0.556143
	0.6	0.781671	0.781671
	0.8	0.939640	0.939640
1.5	0.2	0.276432	0.276432
	0.4	0.537752	0.537752
	0.6	0.765249	0.765249
	0.8	0.932471	0.932471
2.5	0.2	0.267791	0.267791
	0.4	0.524045	0.524044
	0.6	0.752605	0.752605
	0.8	0.926703	0.926702

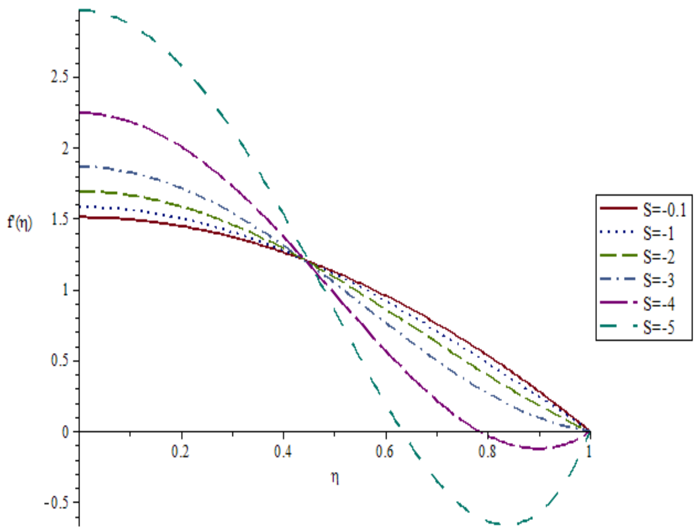

Figure 2:The influence of negative S on f'(η) for Axisymmetric case

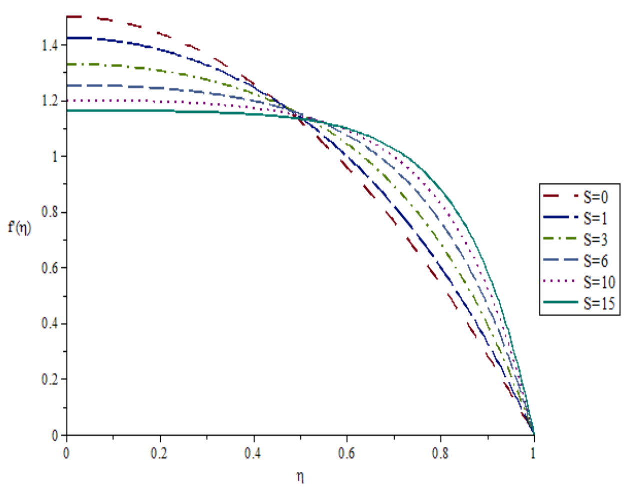

Figure 3: The influence of positive S on f'(η) for two dimensional case

IV. Discussion of results

The method of weighted residual is used to obtain the numerical solution of the fourth order boundary value problem and the residual $R(\eta)$ was minimized by Galerkin method to get series solution. The solution was evaluated at a point in the region in order to compare our results with the literature to check the efficiency of the method as shown in table 1 for axisymmetric case and table 2 for two dimensional case respectively. The effect of negative and positive squeeze number was also shown graphically in fig 2 and fig 3.

V. Conclusion

In this paper, weighted residual method has been successfully employed to solve the fourth order boundary value of fluid problem. The effect of squeeze number that characterized the equation was shown graphically and in tabular form in comparison with Runge-Kutta method which shows the accuracy of the method in solving nonlinear problems.

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