BOWEN UNIVERSITY, IWO, NIGERIA

COLLEGE OF AGRICULTURE, ENGINEERING AND SCIENCE

MATHEMATICS PROGRAMME

B.Sc DEGREE 2022/2023 SECOND SEMESTER EXAMINATION

COURSE CODE: MAT 204 **COURSE TITLE**: MECHANICS

DATE: 26/06/2023 **TIME ALLOWED**: $2\frac{1}{2}$hours **CREDITS**: 3

INSTRUCTION: Attempt any FOUR questions.

1a. A 50 N constant force is exerted on a $10kg$ object for 5 seconds. The initial velocity of the object is $8ms^{-1}$ and the direction of the object is the same as the direction of the force and distance is $105m$.
Compute:
(i) The work done on the object.
(ii) The final K.E.
(iii) The increase in the K.E of the object. (7.5 marks)

b. The position of a particle is given by $x = 150t^2 - 8t^3$. Compute its velocity and acceleration.
marks)

c. A ball is thrown with initial speed $60ms^{-1}$ at an angle of 30^o to the horizontal. What is the horizontal range R, the maximum height H and the time of flight T of the projectile (Take $g = 10ms^{-2}$, neglect air resistance). (6 marks)

2a. A steel wire $8m$ long and $2m$ in diameter is attached to the ceiling and a $100N$ weight is attached to the end. The steel wire stretches $2.05m$. What is:
(i) The applied stress?
(ii) The longitudinal strain?
(iii) The modulus of elasticity for the steel. (8 marks)

b. From the Newton's second law of motion, show the Work-Energy theorem, $W_{net} = K_f - K_i$. (5 marks)

c. Define a couple. Give two examples of a couple. (4.5 marks)

3a. Find the resultant moment that two forces $F_1 = 60\hat{i} - 40\hat{j} + 30\hat{k}N$, $F_2 = 70\hat{i} + 40\hat{j} - 20\hat{k}N$ create by acting on a rod about the point O ($\vec{r_A} = \{3\hat{j} - 2\hat{k}\}m$, $\vec{r_B} = \{4\hat{i} + 2\hat{j} - 2\hat{k}\}m$). (8 marks)

b. A particle of 5 kg has a velocity of $(-3i + 7j)ms^{-1}$. What is its x and y components of momentum and magnitude of its total momentum. (5.5 marks)

c. State two principles of virtual work. (4 marks)

4a. Suppose a particle moves along a helix H with position at time t given by $f(t) = 2\cos\ t\ i + 2\sin\ t\ j$. What is:
(i) the velocity.
(ii) the acceleration of the particle.
(iii) the component of the acceleration in the direction of $T(t)$.
(iv) the component of the acceleration in the direction of $N(t)$. (12 marks)

b. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = 5\hat{i} + 2\hat{j} - 4\hat{k}$.
Compute $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. (5.5 marks)

5a. Find the scalar and vector product of vectors $\vec{a}$ and $\vec{b}$ where $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 2\hat{k}$.
Determine the angle between them. (8.5 marks)

b. A particle is projected at an angle with initial velocity u so as to pass through a point with coordinates (x, y). Show that the equation of the path is

$$y = x\tan\alpha - \frac{gx^2(1 + \tan 2\alpha)}{2u^2}.$$ (4 marks)

c. A mass m is shot vertically upward from the surface of the earth with initial speed v_0. Assuming that the only force is gravity, express v_0 as a function of G, r, M and m. The force on m is $F = \frac{-GMm}{r^2}$. (5 marks)

6a. A particle moves along a space curve $r = (t^3 - 4t)\hat{i} + (t^2 + 4t)\hat{j} + (8t^2 - 3t^3)\hat{k}$ where t is the time.
Find (i) its velocity and acceleration at any time t.
(ii) the magnitude of its velocity and acceleration at $t = 3s$. (8.5 marks)

b. A 20 kg object accelerated from $5ms^{-1}$ to $10ms^{-1}$. Determine the net work done on the object. (4 marks)

c. Let $\vec{a} = 4\hat{i} - 5\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} - 10\hat{j} - 7\hat{k}$ and $\vec{c} = 5\hat{i} + 7\hat{j} - 4\hat{k}$.
Deduce that $a \times (b \times c) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$. (5 marks)