BOWEN UNIVERSITY, IWO, NIGERIA

COLLEGE OF AGRICULTURE, ENGINEERING AND SCIENCE

MATHEMATICS PROGRAMME

B.Sc DEGREE 2022/2023 SECOND SEMESTER EXAMINATION

COURSE CODE: MAT 202 **COURSE TITLE**: ANALYSIS

DATE: 23/06/2023 **TIME ALLOWED**: $2\frac{3}{4}$hours **CREDITS**: 3

INSTRUCTION: Attempt any FOUR questions.

1a. Define (i) a bounded sequence (ii) an unbounded sequence (iii) bounded below sequence (iv) bounded above sequence (v) monotone increasing sequence (vi) monotone decreasing sequence. Give an example in each case. (9 marks)

b. Evaluate the following: (i) $\lim_{n\to\infty} \frac{n}{\sqrt{4n^2+1}}$ (ii) $\lim_{x\to\frac{\pi}{4}} \frac{\sin 2x}{1+\cos 2x}$. (5 marks)

c. Test for convergence of the following series $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$. (3.5 marks)

2a. For what values of x does the series

$$\sum_{n=1}^{\infty} \frac{n(x-1)^n}{2^n(3n-1)} \text{ converges.}$$ (6 marks)

b. Verify Rolle's theorem for the function $f(x) = -x^2 + 6x - 6$ in the interval $1 \leq x \leq 5$. (5.5 marks)

c. Find the Taylor's series expansion of $\cos x$ at $x = 0$ and $x = \frac{\pi}{2}$. (6 marks)

3a. Test for convergence of the following series

$$(i) \sum_{n=1}^{\infty} \frac{1}{n^n} \quad (ii) \sum_{n=1}^{\infty} \frac{1}{(1+n^3)}.$$ (7 marks)

b. Using Taylor's expansion, expand the function $f(x,y) = xe^y$ about the point $(1,0)$ neglecting the terms of degree three and higher. (5.5 marks)

c. Write the first four terms of the following sequences: (i) $\frac{n\sqrt{n}}{n+1}$ (ii) $\frac{2^{n1}}{n^2}$. (5 marks)

4a. If $f(x,y) = \sin^2 x \cos y + \frac{x}{y^2}$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. (6 marks)

b. Check if the following series converges or diverges
(i) $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \frac{1}{4\times5} + \ldots$ (ii) $U_n = \frac{1}{2^{n+2}}$. (Use the comparison test) (6 marks)

c. Verify Mean Value theorem for the function $f(x) = x^3 - 5x^2 - 3x$ in the interval $[1,3]$ where $a = 1$, $b = 3$. (5.5 marks)

5a. Find the range of values of x for which the following series is absolutely convergent

$$\frac{x}{2.5} - \frac{x^2}{3.5^2} + \frac{x^3}{4.5^3} - \frac{x^4}{5.5^4} + \cdots = \sum_{n=1}^{\infty} \frac{x^n}{(n+1)5^n}.$$ (7 marks)

b. State the Rolle's and Mean Value Theorems. (4 marks)

c. A sequence has its nth term given by $U_n = \frac{2n+3}{5n+2}$.
(i) Write the 1st, 5th, 10th, 100th, 1000th, 10,000th and 100,000th terms of the sequence in decimal form. Make a guess as to the limit of this sequence as $n \to \infty$.
(ii) Using the definition of limit, verify that the guess in (i) is actually correct. (6.5 marks)

6a. Discuss the continuity of the functions (i) $f(x) = \cos \pi x$ at $x = 2$
(ii) $f(x) = \frac{3x^2-4x+1}{x^2-3x+2}$. (7 marks)

b. Determine if $f(x, y) = x^2 + 2y^2 - 2x + 4y - 6$ is commutative or not. (5 marks)

c. Compute the sum of the geometric series

$$\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n.$$ (5.5 marks)