BOWEN UNIVERSITY, IWO
COLLEGE OF AGRICULTURE, ENGINEERING AND SCIENCE
MATHEMATICS PROGRAMME
B.Sc DEGREE 2022/2023 SECOND SEMESTER EXAMINATION

COURSE CODE: MAT 228
COURSE TITLE: LINEAR ALGEBRA
DATE: 21/06/2023 **TIME ALLOWED:** $2\frac{1}{2}$ HOURS **CREDITS:** 3

INSTRUCTIONS: Attempt any **four** questions.

1. (a) Define the following:
   (i) vector space (ii) linear combination (6 marks)

   (b) Determine whether the following sets are vector space under the given operations.

   $$(x,\ y,\ z) + (x', y', z') = (x + x',\ y + y',\ z + z'),$$
   $$K(x,\ y,\ z) = (Kx,\ y,\ z).$$
   (6 *marks*)

   (c) For which value of k will the vector $u = (1, 2, k) \in \mathbb{R}^2$ be linear combination of the vector $V = (3, 0, -2)$ and $W = (2, -1, 5)$. (5.5 marks)

2. (a) Define the following:
   (i) Basis (ii) Dimension (2 marks)

   (b) Find the Basis and Dimension of the following homogeneous system: (10 marks)

   $$x + 2y + 2z - s + 3t = 0$$
   $$x + 2y + 3z + s + t = 0$$
   $$3x + 6y + 8z + s + 5t = 0$$

   (c) Define linear dependence and independence of a vector. Hence, determine whether $(1, 2, -3)$, $(1, -3, 2)$, $(2, -1, 5)$ in $\mathbb{R}^3$ are linearly dependent or not. (5.5 marks)

3. (a) Define a linear transformation. Hence, let T be the function that maps $\mathbb{R}^2$, such that $T : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x, y) = (x^2 + y^2, 2xy)$. Prove or disprove that T is a linear transformation. (10.5 marks)

   (b) Define a singular matrix. Hence verify whether matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 6 \\ 1 & 5 & 3 \end{pmatrix}$ is singular or not. (3 marks)

   (c) Define the following: (i) Linear combination (ii) Spanning sets (4 marks)

4. (a) Let W be the subspace of $\mathbb{R}^5$ spanned by $u_1 = (1, 2, -1, 3, 4)$, $u_2 = (2, 4, -2, 6, 8)$, $u_3 = (1, 3, 2, 2, 6)$, $u_4 = (1, 4, 5, 1, 8)$, $u_5 = (2, 7, 3, 3, 9)$. Find a subset of the vectors that form a basis of W. (7 marks)

   (b) Reduce the following matrix to a lower triangular matrix and hence obtain its determinant: (7.5 marks)

   $$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 0 & 4 & 2 \\ 3 & 2 & 0 & 4 \\ 2 & 4 & 2 & 5 \end{pmatrix}$$

(c) Prove that if any two rows (or columns) of square matrix A are identical, then $det(A) = 0$. (3 marks)

5. (a) Prove that if two rows (or columns) of a square matrix A are interchanged to produce matrix B, then the $det(B) = -det(A)$. (7 marks)

   (b) For every square matrix A, Prove that $|A^T| = |A|$. (4 marks)

   (c) Find the inverse of the matrix (6.5 marks)

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 0 \\ 3 & 0 & 1 \end{pmatrix}.$$

6. (a) Define the following: (i) Eigenvalues (ii) Eigenvector (iii) Spectrum (iv) Spectral radius. (4 marks)

   (b) Given that the eigenvalues of matrix $A = \begin{pmatrix} 2 & 4 \\ 3 & 3 \end{pmatrix}$ are $\lambda = 6$ and $\lambda = -1$, find the corresponding eigenvectors of A. (3 marks)

   (c) Given the matrix $A = \begin{pmatrix} 2 & 3 & -2 \\ 1 & 4 & -2 \\ 2 & 10 & -5 \end{pmatrix}$. Find the eigenvalues and eigenvectors of A. (10.5 marks)